

*A new modeling approach  
for DACs and SACs regions  
in the atmospheres of hot emission stars*

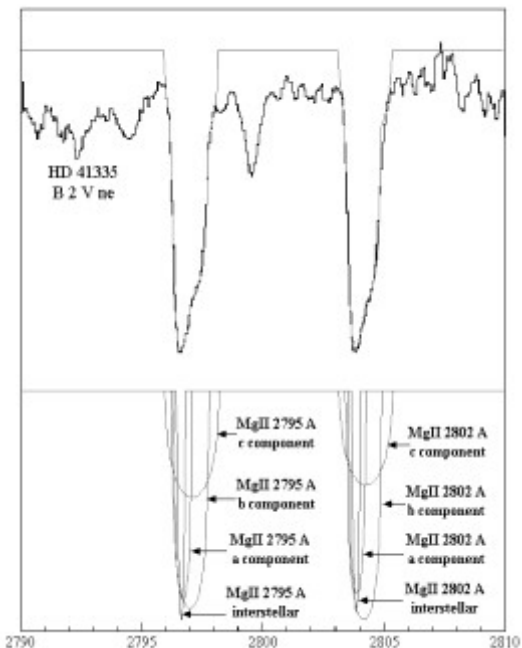
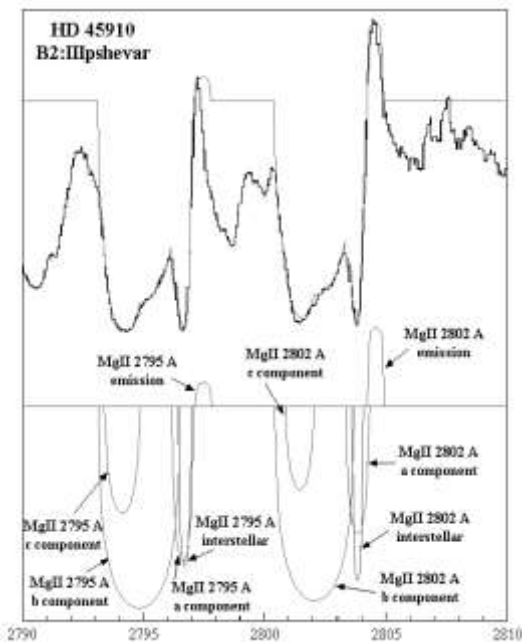
*\*Danezis E., \*Nikolaidis D., \*Lyratzi E, \*\*Popović L. Č.,  
\*\*Dimitriević M. S., \*Theodossiou E., \*Antoniou A.*

*\*University of Athens, School of Physics, Department of Astrophysics,  
Astronomy and Mechanics, Panepistimiopolis, Zografos 157 84,  
Athens – Greece*

*\*\* Astronomical Observatory, Volgina 7, 11160  
Belgrade – Serbia*

## *The DACs and SACs phenomena*

- **DACs** are discrete but not unknown absorption spectral lines. They are spectral lines of the same ion and the same wavelength as a main spectral line, shifted at different  $\Delta\lambda$ , as they are created in different density regions which rotate and move radially with different velocities (Danezis et al. 2003a).
- **DACs** are lines, easily observed, in the spectra of Be stars of luminosity class III, because the regions that give rise to such lines, rotate with low velocities and move radially with high velocities.
- However, if the regions that give rise to such lines rotate with large velocities and move radially with small velocities, the produced lines are much broadened and little shifted.
- As a result they are blended among themselves as well as with the main spectral line and thus they are not discrete. In such a case the name Discrete Absorption Component is inappropriate and we use only the name **SACs (Satellite Absorption Components)**.



- *The best fit is not just the graphical composition of some distribution functions. The reproduced feature is the result of the final function of the model.*
- *The MgII line profile of the star AXMon (HD 45910), which presents DACs and the star HD 41335, which presents SACs are produced in the same way.*
- *The only difference between them is that the components of HD 41335 are much less shifted and thus they are blended among themselves.*
- *The thick line presents the observed spectral line's profile and the thin one the model's fit.*
- *The differences between the observed spectrum and its fit are some times hard to see, as we have accomplished the best fit.*
- *We also present all the components which contribute to the observed features, separately.*

## *The line function*

- *Some years ago our group proposed a new model to explain the complex structure of the density regions of hot stars, where the spectral lines that present SACs or DACs are created*
- *(Danezis et al. 1991, 1998, 2000a,b, 2002a, 2002b, 2003a), Laskarides et al. 1992a, 1992b),*
- *The main hypothesis of this model is that the stellar envelope is composed of a number of successive independent absorbing density layers of matter, a number of emission regions and an external general absorption region*
- *By solving the equations of radiation transfer through a complex structure, as the one described, we conclude to a function for the line's profile, able to give the best fit for the main spectral line and its Satellite Components in the same time.*

*The line profile function is the following:*

$$\bullet I_{\lambda} = \left[ I_{\lambda 0} \prod_i \exp \{-x_{ai}\} + \sum_j S_{\lambda ej} (1 - \exp \{-x_{ej}\}) \right] \exp \{-x_g\} \quad (1)$$

- *where:*
- $I_{\lambda 0}$ : *is the initial radiation intensity,*
- $e^{-x_{ai}}$ ,  $e^{-x_{ej}}$ ,  $e^{-x_g}$ : *are the distribution functions of the absorption, emission and general absorption lines respectively.*
- *This function  $I_{\lambda}$  does not depend on the geometry of the regions which create the observed feature*

## *The rotation model*

- *One of the main hypotheses when we constructed the old version of the model (rotation model), was that the line's width is only a rotational effect and we considered spherical symmetry for the independent density regions.*
- *This means that the random velocities were very low and they did not contribute to the line's broadening.*
- *In this case the above function is transformed and we take the following function:*

$$I_{\lambda} = \left[ I_{\lambda 0} \prod_i \exp \{ -L_i \xi_i \} + \sum_j S_{\lambda e j} (1 - \exp \{ -L_{e j} \xi_{e j} \}) \right] \exp \{ -L_g \xi_g \} \quad (2)$$

$$I_{\lambda} = \left[ I_{\lambda 0} \prod_i \exp \{ -L_i \xi_i \} + \sum_j S_{\lambda ej} (1 - \exp \{ -L_{ej} \xi_{ej} \}) \right] \exp \{ -L_g \xi_g \} \dots (2)$$

- *where:*
- $I_{\lambda 0}$ : *is the initial radiation intensity,*
- $L_i$   $L_{ej}$   $L_g$ : *are the distribution functions of the absorption*
- *coefficients*  $k_{\lambda i}$   $k_{\lambda ej}$   $k_{\lambda g}$ .
- $\xi$  *is the optical depth,*
- $S_{\lambda ej}$ : *is the source function, which, at the moment when the spectrum is taken, is constant.*

- In this case we can calculate that **L** is the following function:

$$L(\lambda) = \int_{-\theta_0}^{\theta_0} R \cos \theta \left[ \frac{\arctan[\lambda - \lambda_{lab}(1 - z_0 \cos \theta)] - \arctan[\lambda - \lambda_{lab}(1 + z_0 \cos \theta)]}{\lambda_{lab} z_0 \cos \theta} \right] d\theta \dots (.3)$$

- where **R** is the radius of the spherical density region,  **$\lambda_{lab}$**  is the laboratory wavelength of the spectral line produced by a particular ion and,

$$z_0 = \frac{\Delta \lambda_{rotation}}{\lambda_{lab}} = \frac{V_{rotation}}{c}$$

- where  **$\Delta \lambda_{rotation}$**  is the width and  **$V_{rotation}$**  the apparent rotational velocity of the *i* density shell of matter.



$$L(\lambda) = \int_{-\theta_0}^{\theta_0} R \cos \theta \left[ \frac{\arctan[\lambda - \lambda_{lab}(1 - z_0 \cos \theta)] - \arctan[\lambda - \lambda_{lab}(1 + z_0 \cos \theta)]}{\lambda_{lab} z_0 \cos \theta} \right] d\theta \dots (.3)$$

- *The polynomial*

$$P(\lambda) = \frac{\arctan[\lambda - \lambda_{lab}(1 - z_0 \cos \theta)] - \arctan[\lambda - \lambda_{lab}(1 + z_0 \cos \theta)]}{\lambda_{lab} z_0 \cos \theta}$$

can be well approximated by the polynomial  $f(\lambda)$ , where

$$f(\lambda) = \begin{cases} 1, & \text{if } |\lambda - \lambda_0| < \rho \\ 0, & \text{if } |\lambda - \lambda_0| \geq \rho \end{cases} \quad \text{where} \quad \rho = \frac{\lambda_0 z_0 \cos \theta}{1 - z_0^2 \cos^2 \theta}$$

$$\rho = \frac{\lambda_0 z_0 \cos \theta}{1 - z_0^2 \cos^2 \theta}$$

- $\lambda_0$  is the observed wavelength of the center of the spectral line,

$$\lambda_0 = \lambda_{lab} + \Delta\lambda_{rad}$$

- and  $\Delta\lambda_{rad}$  is the radial Doppler shift

$$\frac{\Delta\lambda_{rad}}{\lambda_{lab}} = \frac{V_{rad}}{c}$$

- Thus, for  $|\lambda - \lambda_0| < \rho = \frac{\lambda_0 z_0 \cos \theta_0}{1 - z_0^2 \cos^2 \theta_0}$
- and if,  $\theta \leq \theta_0$
- then  $P(\lambda)=1$
- and the distribution  $L(\lambda)$  yields:

$$L(\lambda) \cong \int_{-\theta_0}^{\theta_0} R \cos \theta d\theta$$

- We normalize for  $R=1$  and thus

$$L(\lambda) \cong \int_{-\theta_0}^{\theta_0} \cos \theta d\theta = [\sin \theta]_{-\theta_0}^{\theta_0} = 2 \sin \theta_0$$

- We normalize and we have

$$L(\lambda) \cong \sin \theta_0 = \sqrt{1 - \cos^2 \theta_0}$$

By solving the equation  $\Delta\lambda_{rotation} = \frac{\lambda_0 z_0 \cos \theta_0}{1 - z_0^2 \cos^2 \theta_0}$  we have

$$\cos \theta_0 = \frac{-\lambda_0 \pm \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation}z_0}$$

As  $\theta_0$  lies between  $-\pi/2$  and  $\pi/2$ ,  $\cos\theta_0$  must be  $\geq 0$ . So

$$\cos \theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation}z_0}$$

- If  $\cos \theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation}z_0} < 1$  then

$$L(\lambda) = \sqrt{1 - \cos^2 \theta_0}$$

- if  $\cos \theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation}z_0} \geq 1$  then

$$L(\lambda) = 0$$

- The  $L(\lambda)$  is a function of  $V_{rot}$  (that we can take from the  $\Delta\lambda_{rotation}$ ) and  $V_{rad}$  (That we can take from the  $\lambda_0$ )

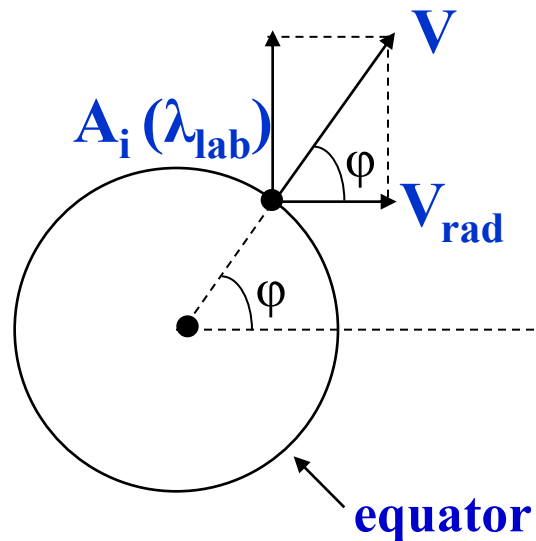
- *Finally the spectral line's profile, which is formed by the  $i$  density shell of matter, must be accurately reproduced by the function by applying the appropriate values of  $V_{roti}$ ,  $V_{radi}$  and  $\xi_i$*
- *Using the best model's fit for a complex spectral line, we can calculate the apparent radial velocity ( $V_{radi}$ ), the apparent rotational velocity ( $V_{roti}$ ) and the optical depth ( $\xi_i$ ) of the region in which the main spectral line and its **SACs** are created.*

## *The new modeling approach*

*In the present work we propose a new approach of the problem, as we also consider the parameter of random velocities in the calculation of the distribution function  $\mathbf{L}$ . This new  $\mathbf{L}$  is a synthesis of the rotational distribution  $\mathbf{Lr}$  that we had presented in the old rotational model and a Gaussian. This means that the new  $\mathbf{L}$  has two limits, the first one gives us a Gaussian and the other the old rotational  $\mathbf{Lr}$ .*

## *The new calculation of the distribution functions $L$*

*Let us consider a spherical shell and a point  $A_i$  in its equator.  
If the laboratory wavelength of a spectral line that arises from  $A_i$  is  $\lambda_{lab}$ , the observed wavelength will be  $\lambda_0 = \lambda_{lab} + \Delta\lambda_{rad}$*





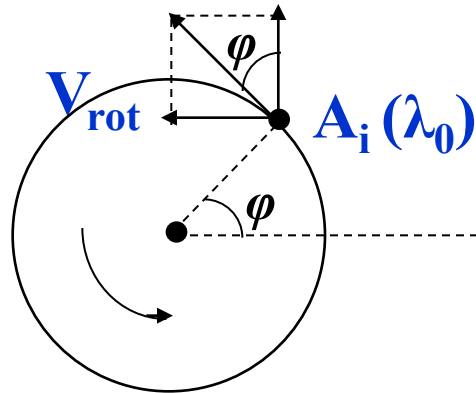
## The new calculation of the distribution functions $L$

If the spherical density region rotates, we will observe a displacement  $\Delta\lambda_{rot}$  and the new wavelength of the center of the line  $\lambda_i$

is:  $\lambda_i = \lambda_0 \pm \Delta\lambda_{rot}$  where  $\Delta\lambda_{rot} = \lambda_0 z \sin \varphi$

$$z = \frac{V_{rot}}{c} = \frac{\Delta\lambda_{rot}}{\lambda_0 \sin \varphi}$$

where  $V_{rot}$  is the observed rotational velocity of the point  $A_i$ .



This means that  $\lambda_i = \lambda_0 \pm \lambda_0 z \sin \varphi = \lambda_0 (1 \pm z \sin \varphi)$

and if  $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$  then  $\lambda_i = \lambda_0 (1 - z \sin \varphi)$

*If we consider that the spectral line profile is a Gaussian distribution we have:*

$$P(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{\lambda - \kappa}{\sigma\sqrt{2}}\right]^2}$$

*where  $\kappa$  is the mean value of the distribution and in the case of the line profile it indicates the center of the spectral line that arises from  $A_i$ . This means that:*

$$P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{\lambda - \lambda_0(1 - z \sin \varphi)}{\sigma\sqrt{2}}\right]^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[\lambda - \lambda_0(1 - z \sin \varphi)]^2}{2\sigma^2}}$$

for all the semi-equator we have

$$L(\lambda) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{[\lambda - \lambda_0(1 - z \sin \varphi)]^2}{2\sigma^2}} \cos \varphi d\varphi \dots \dots (4)$$

If we make the transformation,  $\sin \varphi = x$  and  $u = \frac{\lambda - \lambda_0(1 - zx)}{\sigma\sqrt{2}}$

the above function (4) will be transformed and finally we have the function (5) :

$$L(\lambda) = \frac{1}{\lambda_0 z \sqrt{\pi}} \int_{\frac{\lambda - \lambda_0(1+z)}{\sigma\sqrt{2}}}^{\frac{\lambda - \lambda_0(1-z)}{\sigma\sqrt{2}}} e^{-u^2} du$$

$$L(\lambda) = \frac{1}{\lambda_0 z \sqrt{\pi}} \left[ \int_0^{\frac{\lambda - \lambda_0(1-z)}{\sigma\sqrt{2}}} e^{-u^2} du - \int_0^{\frac{\lambda - \lambda_0(1+z)}{\sigma\sqrt{2}}} e^{-u^2} du \right] \dots (5)$$

The above integrals have the form of a known integral **erf(x)**

$$\operatorname{erf}(x) = \frac{2}{\pi} \int_0^x e^{-u^2} du$$

that has the following properties:

1.  $\operatorname{erf}(x) = -\operatorname{erf}(-x)$
2.  $\operatorname{erf}(0) = 0$
3.  $\operatorname{erf}(+\infty) = 1$  as  $\lim_{x \rightarrow +\infty} \operatorname{erf}(x) = 1$
4.  $\operatorname{erf}(-1) = -1$
5. 
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3 \cdot 1!} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \dots \right)$$
6. 
$$\operatorname{erf}(x) = 1 - \frac{e^{-x^2}}{\sqrt{\pi}x} \left( 1 - \frac{1}{2x^2} + \frac{1 \cdot 3}{(2x^2)^2} - \frac{1 \cdot 3 \cdot 5}{(2x^2)^3} + \dots \right)$$

$$L(\lambda) = \frac{\sqrt{\pi}}{2\lambda_0 z} \left[ \operatorname{erf} \left( \frac{\lambda - \lambda_0(1-z)}{\sqrt{2}\sigma} \right) - \operatorname{erf} \left( \frac{\lambda - \lambda_0(1+z)}{\sqrt{2}\sigma} \right) \right] \quad (6)$$

*The distribution function from the semi-spherical region is:*

$$L_{final}(\lambda) = \frac{\sqrt{\pi}}{2\lambda_0 z} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[ \operatorname{erf} \left( \frac{\lambda - \lambda_0}{\sqrt{2}\sigma} + \frac{\lambda_0 z}{\sqrt{2}\sigma} \cos \theta \right) - \operatorname{erf} \left( \frac{\lambda - \lambda_0}{\sqrt{2}\sigma} - \frac{\lambda_0 z}{\sqrt{2}\sigma} \cos \theta \right) \right] \cos \theta d\theta \quad (7)$$

*(Method Simpson)*

This  $L_{final}(\lambda)$  is the distribution that replaces the old rotational distribution **L** that our group proposed some years ago (Danezis et al 2001).

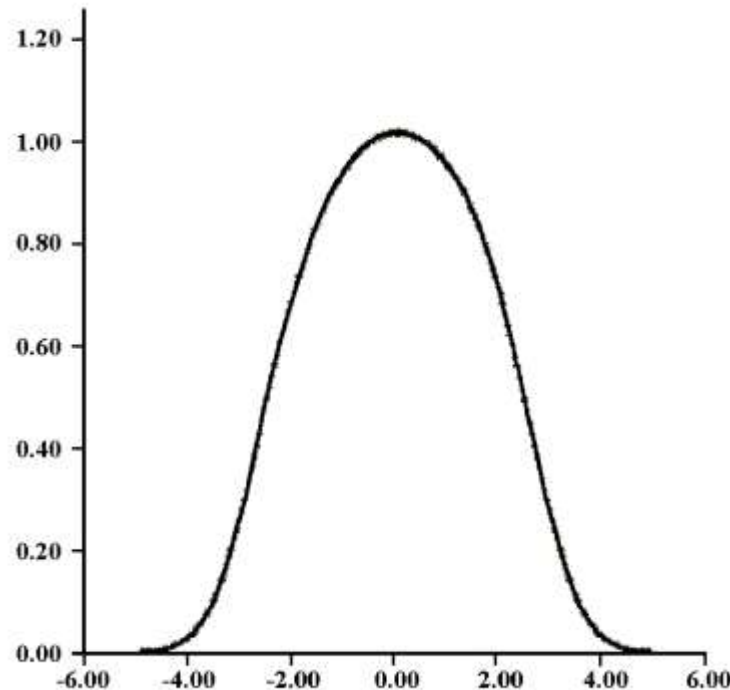
## *Discussion*

*In the proposed distribution an important factor is  $m = \frac{\lambda_0 z}{\sqrt{2}\sigma}$*

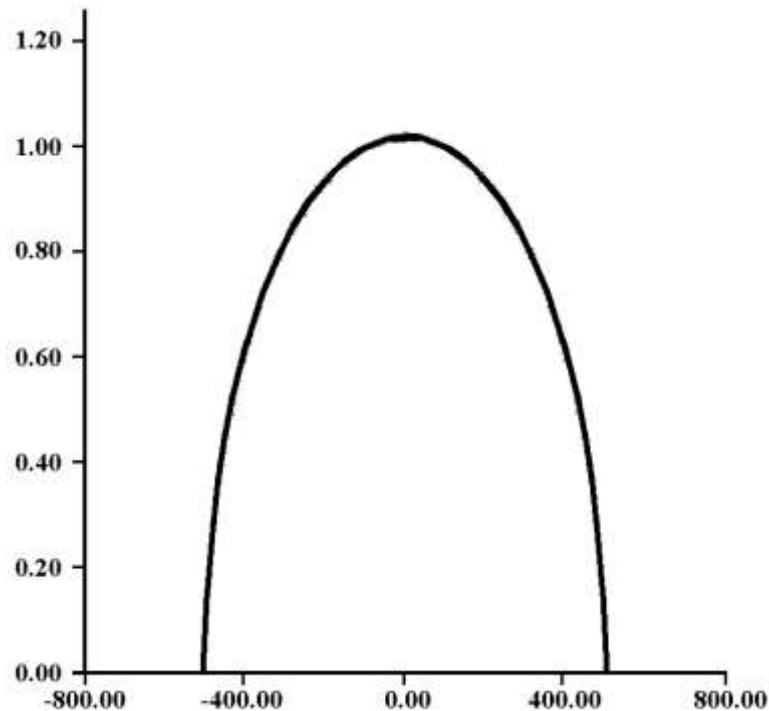
*This factor indicates the kind of the distribution that can fit the line profile.*

*1. If  $m \cong 3$  we have a mixed distribution. The line broadening is an effect of two equal reasons:*

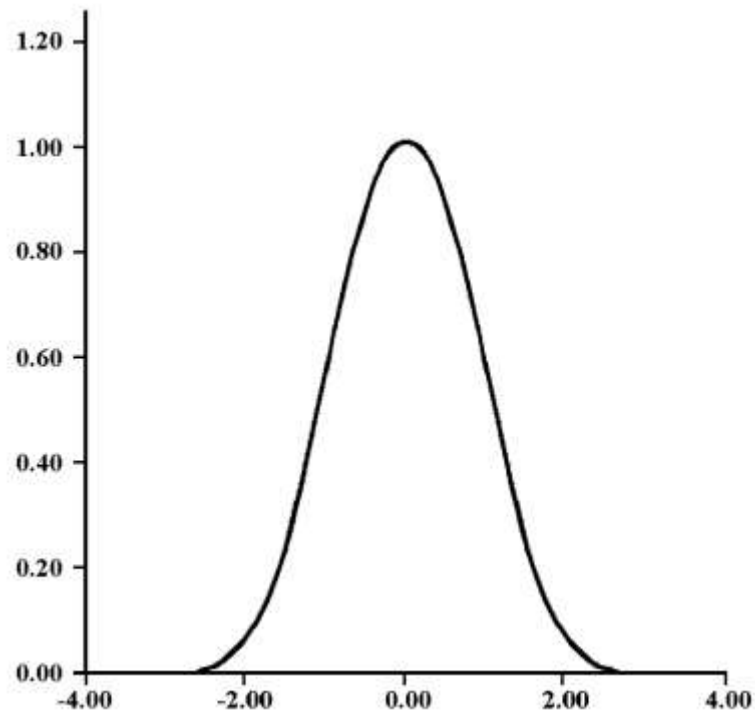
- a. The rotational velocity of the spherical region and*
- b. The random velocities of the ions.*



2. If  $m \cong 500$  *the line broadening is only an effect of the rotational velocity* and the random velocities are very low. In this case the profile of the line is the same with the profile that we can produce using the old rotation model (Danezis 2001, 2003).



3. Finally, if  $m < 1$  *the line broadening is only an effect of random velocities and the line distribution is a Gaussian.*





## *The column density*

*An important point of our study is the calculation of the column density from our model.*

*Lets start from the definition of the optical depth:  $\tau = \int_0^s k\rho ds$*   
*where  $\tau$  is the optical depth (no units),*

*$k$  is the absorption coefficient (  $\frac{cm^2}{gr}$  ),*

*$\rho$  is the density of the absorbing region (  $\frac{gr}{cm^3}$  ),*

*$s$  is the geometrical depth (cm)*

In the model we set  $k = L\Omega$ , so  $\tau = \int_0^s L\Omega\rho ds$   
where  $L$  is the distribution function of the absorption coefficient  $k$   
and has no units,

$\Omega$  equals 1 and has the units of  $k$  ( $\Omega = 1 \frac{cm^2}{gr}$ )

We consider that for the moment of the observation and for a  
significant ion,  $k$  is constant, so  $k$  (and thus  $L$  and  $\Omega$ ) may come out

of the integral. So:  $\tau = L \int_0^s \Omega\rho ds$

We set  $\xi = \int_0^s \Omega\rho ds$

and  $\tau$  becomes  $\tau = L\xi$

## Absorption lines

For every one of  $\xi$  along the spectral line (henceforth called  $\xi_i$ )

we have that: 
$$\xi_i = \int_0^s \Omega \rho ds \Rightarrow \xi_i = \Omega \int_0^s \rho ds \Rightarrow \frac{\xi_i}{\Omega} = \int_0^s \rho ds$$

We set 
$$\sigma_i = \frac{\xi_i}{\Omega} = \int_0^s \rho ds$$

As  $\Omega \cdot \left(1 \frac{\text{cm}^2}{\text{gr}}\right)$  contributes only to the units,  $\sigma_i$  takes

the value of  $\xi_i$  .

For each of  $\lambda_i$  along the spectral line, we extract a  $\sigma_i$  from each  $\xi_i$  . The program we use calculates the  $\xi_i$  for the centre of the line. This means that from this  $\xi_i$  we can measure the respective  $\sigma_i$  .

If we add the values of all  $\sigma_i$  along the spectral line then we have

$$\sigma = \sum_i \sigma_i \quad \left( \frac{\text{gr}}{\text{cm}^2} \right),$$

which is the **surface density** of the absorbing matter, which creates the spectral line.

If we divide  $\sigma$  with the **atomic weight** of the ion which creates the spectral line, we extract the **number density of the absorbers**, meaning the number of the absorbers per square centimetre

$$\left( n = \frac{\sigma}{AW} \quad (\text{in } \text{cm}^{-2} \text{ })). \right)$$

*This number density corresponds to **the energy density** which is absorbed by the whole matter which creates the observed spectral*

*line (  $\frac{E}{AW}$  (in  $\frac{\text{erg}}{\text{cm}^2}$  )) and which **is calculated by the model.***

*It is well known, that each absorber absorbs the specific amount of the energy needed for the transition which creates the specific line.*

*This means that if **we divide the calculated energy density***

*(  $\frac{E}{AW}$  ) **with the energy needed for the transition, we obtain***

***the column density (in  $\text{cm}^{-2}$  ).***

## Emission lines

In the case of the emission lines we have to take into account not only  $\xi_e$ , but also the source function  $S$ , as both of these parameters contribute to the height of the emission lines. So in this case we have:

$$S\xi_e = \frac{j}{k} \int_0^s \Omega \rho_e ds$$

where:  $j$  is the emission coefficient (  $\frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot A}$  ),

$k$  is the absorption coefficient (  $\frac{\text{cm}^2}{\text{gr}}$  )

$\rho_e$  is the density of the emitting region (  $\frac{\text{gr}}{\text{cm}^3}$  )

$s$  is the geometrical depth (cm)

We set  $k = L\Omega$

where  $L$  is the distribution function of the absorption coefficient  $k$  and has no units,

$\Omega$  equals 1 and has the units of  $k$  ( $\Omega = 1 \frac{\text{cm}^2}{\text{gr}}$ )

And  $j = L_e\Omega_e$

where  $L_e$  is the distribution function of the emission coefficient  $j$  and has no units,

$\Omega_e$  equals 1 and has the units of  $j$  ( $\Omega_e = 1 \frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot A}$ )

As we did before, in the case of the absorption lines, we may consider that  $\Omega$  may come out of the integral.

$$\text{So: } S\xi_e = \frac{j}{k} \int_0^s \Omega \rho_e ds = \frac{L_e \Omega_e}{L \Omega} \int_0^s \Omega \rho_e ds = \frac{L_e \Omega_e}{L \Omega} \Omega \int_0^s \rho_e ds = \frac{L_e \Omega_e}{L} \int_0^s \rho_e ds$$

*As in the model we use the same distribution for the absorption and for the emission  $L_e = L$  .*

$$\text{So: } S\xi_e = \Omega_e \int_0^s \rho_e ds \Rightarrow \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$$

$$\text{We set } \sigma_e = \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$$

*As  $\Omega_e..(=1 \frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot \text{A}})$  contributes only to the units,*

*$\sigma_e$  takes the value of  $S\xi_e$ .*



*For each  $\lambda_i$  along the spectral line, we extract a  $\sigma_i$  from each  $S\xi_e$ . The program we use calculates the  $\xi_e$  for the center of the line and the  $S$ . This means that from this  $\xi_e$  and  $S$  we can measure the respective  $\sigma_i$ .*

*If we add the values of all  $\sigma_i$  along the spectral line then we have*

$$\sigma = \sum_i \sigma_i \text{ (in } \frac{\text{gr}}{\text{cm}^2} \text{),}$$

*which is the **surface density** of the emitting matter, which creates the spectral line. If we divide  $\sigma$  with the atomic weight of the ion which creates the spectral line, we extract **the number density** of the emitters, meaning the number of the emitters per square centimetre*

$$n = \frac{\sigma}{AW} \text{ (in } \text{cm}^{-2} \text{).}$$

This **number density** corresponds to the **energy density** which is emitted by the whole matter which creates the observed spectral line

(  $\frac{E}{AW}$  (in  $\frac{\text{erg}}{\text{cm}^2}$  )) and which **is calculated by the model**.

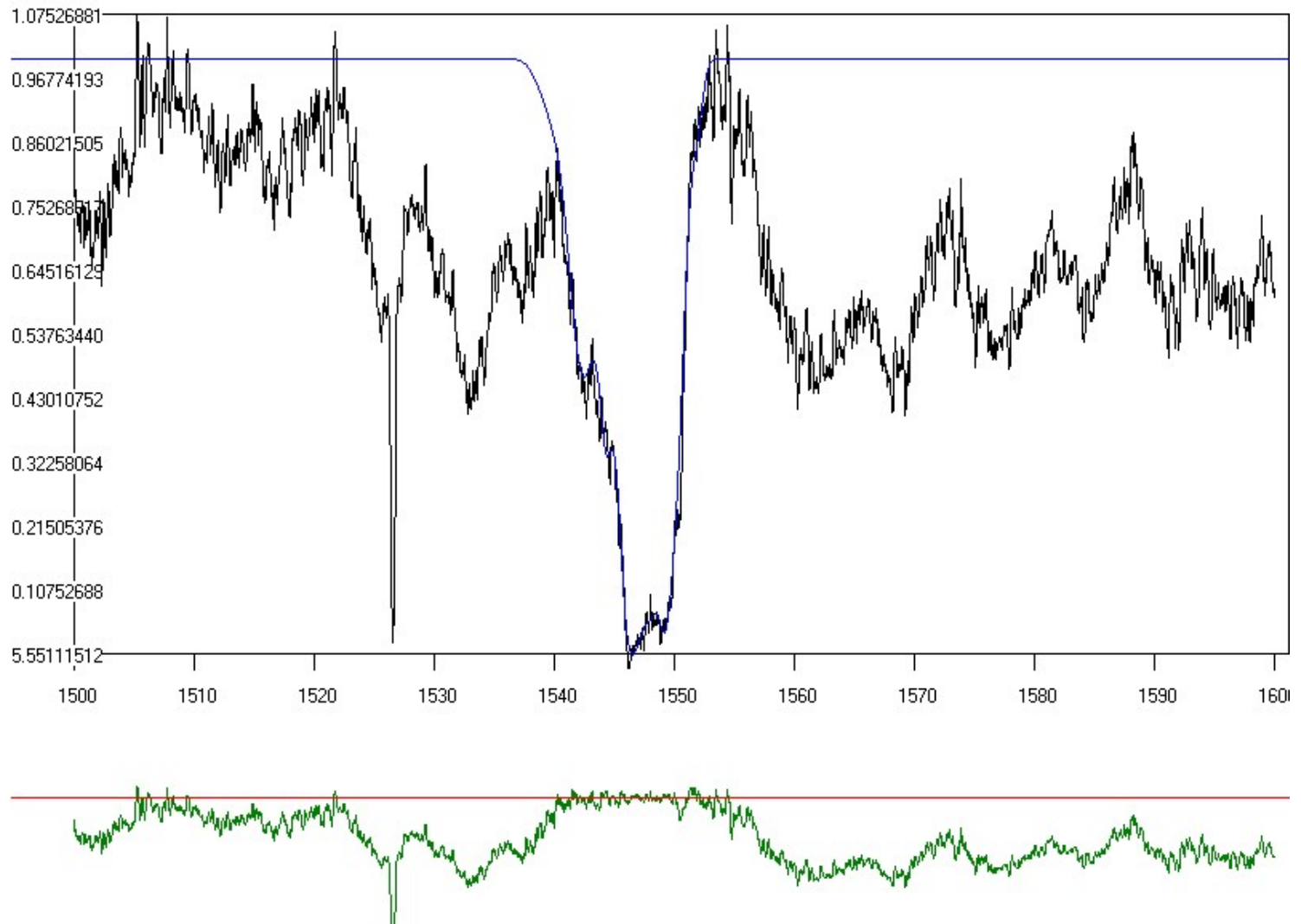
It is well known, that each emitter emits the specific amount of the energy needed for the transition which creates the specific line.

This means that if we divide the calculated energy density(  $\frac{E}{AW}$  )

with the **energy needed for the transition**, we obtain the **column**

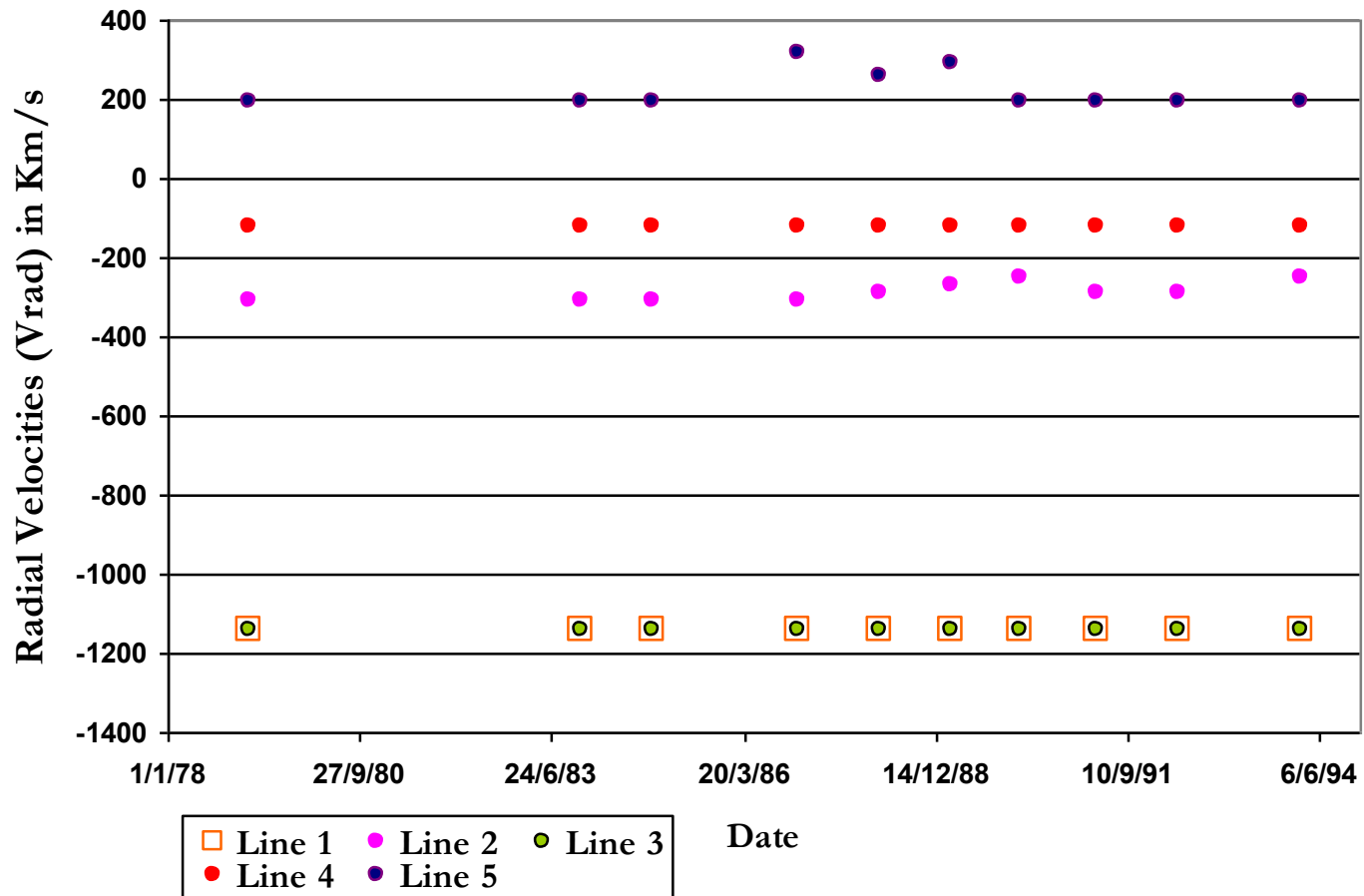
**density** (in  $\text{cm}^{-2}$ ).

# CIV doublet in the spectrum of the star HD93521



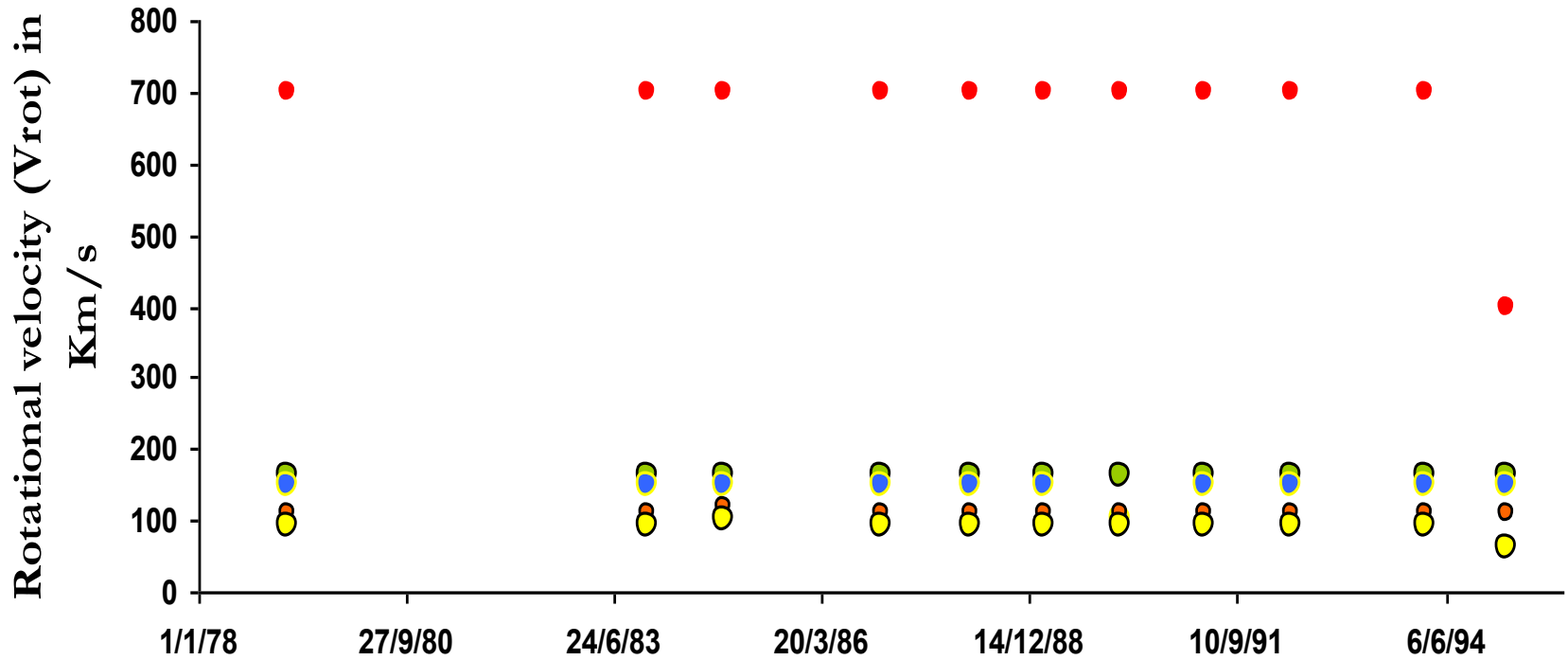
# C IV - REGION OF THE STAR HD 93521

Time-scale changes of the radial velocities ( $V_{rad}$ ) of the 5 independent satellite components



# C IV - REGION OF THE STAR HD 93521

Time-scale changes of the rotational velocity ( $V_{rot}$ ) of the 5 independent satellite components



● Line 1 ● Line 2 ● Line 3 ● Line 4 ● Line 5

**Thank you very much  
for your attention  
and for your hospitality**