

Introduction

As it is already known, the spectra of many Oe and Be stars present Discrete Absorption Components (DACs) which, due to their profiles' width as well as the values of the radial velocities, create a complicated profile of the main spectral lines. This fact is interpreted by the existence of two or more independent layers of matter, in the region where the spectral lines are formed. Such a structure is responsible for the formation of a series of satellite components (DACs or SACs) for each spectral line (Bates & Halliwell, 1986, Danezis et al. 2003, 2005).

In this paper we present a mathematical model reproducing the complex profile of the spectral lines of Oe and Be stars that present DACs or SACs. This model presupposes that the regions, where these spectral lines are formed, are not continuous but consist of a number of independent absorbing or emitting density layers of matter and an external general absorption region. In this model we assume that the line broadening is due to the random motion of the ions and the rotation of the density regions that produce the spectral line and its satellite components. With this method we can calculate the values of the apparent rotational and radial velocities, the Gaussian standard deviation of the random motions of the ions, the random velocities, the absorbed and the emitted energy and finally the column density of the independent regions of matter which produce the main and the satellites components of the studied spectral lines.

In order to check the above spectral line function, we calculated the rotational velocity of Hel λ 4387.928 Å absorption line in the spectra of five Be stars, using two methods, the classical Fourier analysis and our model. The values of the rotational velocities, calculated with Fourier analysis, are the same with the values calculated with our method.

We point out that the new and important aspect of this method is the values' calculation of the above parameters using the DACs or SACs theory.

The DACs and SACs phenomena

DACs are discrete but not unknown absorption spectral lines. They are spectral lines of the same ion and the same wavelength as a main spectral line, shifted at different Δλ, as they are created in different density regions which rotate and move radially with different velocities (Danezis et al. 2003).

DACs are lines, easily observed, when the regions that give rise to such lines, rotate with low velocities and move radially with high velocities.

However, if the regions that give rise to such lines rotate with large velocities and move radially with small velocities, the produced lines are much broadened and little shifted.

As a result they are blended among themselves as well as with the main spectral line and thus they are not discrete. In such a case the name Discrete Absorption Component is inappropriate and we use only the name SACs (Satellite Absorption Components).

The line function

Some years ago our group proposed a new model to explain the complex structure of the density regions of hot stars, where the spectral lines that present SACs or DACs are created (Danezis et al. 1991, 1998, 2000a,b,c, 2002a,b,c, 2003), Laskarides et al. 1992a,b).

The main hypothesis of this model is that the stellar envelope is composed of a number of successive independent absorbing density layers of matter, a number of emission regions and an external general absorption region.

By solving the equations of radiation transfer through a complex structure, as the one described, we conclude to a function for the line's profile, able to give the best fit for the main spectral line and its Satellite Components in the same time.

The line profile function is the following:

$$I_{\lambda} = \left[I_{i0} \prod \exp(-x_{s_j}) + \sum S_{s_j} (1 - \exp(-x_{s_j})) \right] \exp(-x_g) \quad (1)$$

where:

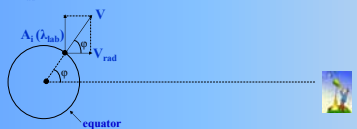
I_{i0} : is the initial radiation intensity, $e^{-x_{s_j}}$, $e^{-x_{s_j}}$, e^{-x_g} : are the distribution functions of the absorption, emission and general absorption lines respectively. This function I_{λ} does not depend on the geometry of the regions which create the observed feature.

The new modeling approach

In the present work we propose a new approach of the problem, as we also consider the parameter of random velocities in the calculation of the distribution function L . This new L is a synthesis of the rotational distribution L_r that we had presented in the old rotational model and a Gaussian. This means that the new L has two limits, the first one gives us a Gaussian and the other the old rotational L_r .

The new calculation of the distribution functions L

Let us consider a spherical shell and a point A_i in its equator. If the laboratory wavelength of a spectral line that arises from A_i is λ_{lab} , the observed wavelength will be $\lambda_{\lambda} = \lambda_{lab} + \Delta\lambda_{rad}$



If the spherical density region rotates, we will observe a displacement $\Delta\lambda_{rot}$ and the new wavelength of the center of the line λ_i is $\lambda_i = \lambda_0 \pm \Delta\lambda_{rot}$ where $\Delta\lambda_{rot} = \lambda_0 z \sin\phi$
 $z = \frac{V_{rot}}{c} = \frac{\Delta\lambda_{rot}}{\lambda_0 \sin\phi}$ where V_{rot} is the observed rotational velocity of the point A_i



This means that $\lambda_i = \lambda_0 \pm \lambda_0 z \sin\phi = \lambda_0 (1 \pm z \sin\phi)$ and if $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$ then $\lambda_i = \lambda_0 (1 - z \sin\phi)$

If we consider that the spectral line profile is a Gaussian distribution we have:

$$P(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[\lambda - \lambda_i]^2}{2\sigma^2}} \quad (2)$$

where λ_i is the mean value of the distribution and in the case of the line profile it indicates the center of the spectral line that arises from A_i . This means that:

$$P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[\lambda - \lambda_0(1 - z \sin\phi)]^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[\lambda - \lambda_0(1 - z \sin\phi)]^2}{2\sigma^2}} \quad (3)$$

for all the semi-equator we have:

$$L(\lambda) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[\lambda - \lambda_0(1 - z \sin\theta)]^2}{2\sigma^2}} \cos\theta d\theta \quad (4)$$

If we make the transformation $\sin\theta = x$ and $u = \frac{\lambda - \lambda_0(1 - zx)}{\sigma\sqrt{2}}$ then

$$L(\lambda) = \frac{1}{\lambda_0 z \sqrt{\pi}} \int_{\frac{\lambda - \lambda_0(1-z)}{\sigma\sqrt{2}}}^{\frac{\lambda - \lambda_0(1+z)}{\sigma\sqrt{2}}} e^{-u^2} du$$

$$\text{or } L(\lambda) = \frac{1}{\lambda_0 z \sqrt{\pi}} \left[\int_0^{\frac{\lambda - \lambda_0(1-z)}{\sigma\sqrt{2}}} e^{-u^2} du - \int_0^{\frac{\lambda - \lambda_0(1+z)}{\sigma\sqrt{2}}} e^{-u^2} du \right] \quad (5)$$

$$L(\lambda) = \frac{\sqrt{\pi}}{2\lambda_0 z} \left[\text{erf}\left(\frac{\lambda - \lambda_0(1-z)}{\sqrt{2}\sigma}\right) - \text{erf}\left(\frac{\lambda - \lambda_0(1+z)}{\sqrt{2}\sigma}\right) \right] \quad (6)$$

The distribution function from the semi-spherical region is:

$$L_{\mu\text{rad}}(\lambda) = \frac{\sqrt{\pi}}{2\lambda_0 z} \int_{\frac{\pi}{2}}^{\pi} \left[\text{erf}\left(\frac{\lambda - \lambda_0}{\sqrt{2}\sigma} + \frac{\lambda_0 z}{\sqrt{2}\sigma} \cos\theta\right) - \text{erf}\left(\frac{\lambda - \lambda_0}{\sqrt{2}\sigma} - \frac{\lambda_0 z}{\sqrt{2}\sigma} \cos\theta\right) \right] \cos\theta d\theta \quad (7)$$

(Method Simpson)

This $L_{\mu\text{rad}}(\lambda)$ is the distribution that replaces the old rotational distribution L that our group proposed some years ago (Danezis et al 2003).

The column density

An important point of our study is the calculation of the column density from our model.

Lets start from the definition of the optical depth: $\tau = \int_0^s k \rho dx$

where τ is the optical depth (no units),

k is the absorption coefficient ($\frac{\text{cm}^2}{\text{gr}}$)

ρ is the density of the absorbing region ($\frac{\text{gr}}{\text{cm}^3}$),

s is the geometrical depth (cm)

In the model we set $k = L\Omega$, so $\tau = \int_0^s L\Omega\rho dx$

where L is the distribution function of the absorption coefficient k and has no units, Ω equals 1 and has the units of k ($\Omega = 1 - \frac{\text{cm}^2}{\text{gr}}$)

We consider that for the moment of the observation and for a significant ion, k is constant, so k (and thus L and Ω) may come out of the integral. So: $\tau = L_0 \int_0^s \Omega\rho dx$

We set $\xi = \int_0^s \Omega\rho dx$ and τ becomes $\tau = L_0 \xi$

Absorption lines

For every one of ξ along the spectral line (henceforth called ξ_i) we have that:

$$\xi_i = \int_0^s \Omega\rho dx \Rightarrow \xi_i = \Omega_i \int_0^s \rho dx \Rightarrow \xi_i = \int_0^s \sigma_i \rho dx$$

We set $\sigma_i = \frac{\xi_i}{\Omega} = \int_0^s \rho dx$

As Ω ($\frac{\text{cm}^2}{\text{gr}}$) contributes only to the units, σ_i takes the value of ξ_i .

For each λ_i along the spectral line, we extract a σ_i from each ξ_i . The program we use calculates the ξ_i for the centre of the line. This means that from this ξ_i we can measure the respective σ_i .

If we add the values of all σ_i along the spectral line then we have:

$$\sigma = \sum \sigma_i \left(\frac{\text{gr}}{\text{cm}^2} \right)$$

which is the surface density of the absorbing matter, which creates the spectral line.

If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the absorbers, meaning the number of the absorbers per square centimetre ($n = \frac{\sigma}{AW}$) (in cm^{-2}).

This number density corresponds to the energy density ($\frac{E}{AW}$) (in $\frac{\text{erg}}{\text{cm}^2}$) which is absorbed by the whole matter which creates the observed spectral line and which is calculated by the model.

It is well known, that each absorber absorbs the specific amount of the energy needed for the transition which creates the specific line.

This means that if we divide the calculated energy density ($\frac{E}{AW}$) with the energy needed for the transition, we obtain the column density (in cm^{-2}).

Emission lines

In the case of the emission lines we have to take into account not only ξ_i , but also the source function S , as both of these parameters contribute to the height of the emission lines. So in this case we have:

$$S \xi_i = \int_0^s \Omega \rho_e ds$$

where: j is the emission coefficient ($\frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot \text{A}}$),

k is the absorption coefficient ($\frac{\text{cm}^2}{\text{gr}}$)

ρ_e is the density of the emitting region ($\frac{\text{gr}}{\text{cm}^3}$)

s is the geometrical depth (cm)

We set $k = L\Omega$

where L is the distribution function of the absorption coefficient k and has no units,

Ω equals 1 and has the units of k ($\Omega = 1 - \frac{\text{cm}^2}{\text{gr}}$)

and $j = L_e \Omega_e$

where L_e is the distribution function of the emission coefficient j and has no units,

Ω_e equals 1 and has the units of j ($\Omega_e = 1 - \frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot \text{A}}$)

As we did before, in the case of the absorption lines, we may consider that Ω may come out of the integral.

$$\text{So: } S \xi_i = \int_0^s \Omega \rho_e ds = \frac{L_e \Omega_e}{L \Omega} \int_0^s \Omega \rho_e ds = \frac{L_e \Omega_e}{L} \int_0^s \rho_e ds$$

As in the model we use the same distribution for the absorption and for the emission, we know that $L_e = L$.

$$\text{So: } S \xi_i = \int_0^s \Omega_e \rho_e ds \Rightarrow \Omega_e = \int_0^s \rho_e ds$$

We set $\sigma_e = \frac{S \xi_i}{\Omega_e} = \int_0^s \rho_e ds$

As $\Omega_e = 1 - \frac{\text{erg}}{\text{gr} \cdot \text{s} \cdot \text{rad} \cdot \text{A}}$ contributes only to the units,

σ_e takes the value of $S \xi_i$.

For each λ_i along the spectral line, we extract a σ_i from each $S \xi_i$. The program we use calculates ξ_i for the center of the line and S . This means that from this ξ_i and S we can measure the respective σ_i .

If we add the values of all σ_i along the spectral line then we have

$$\sigma = \sum \sigma_i \left(\frac{\text{gr}}{\text{cm}^2} \right),$$

which is the surface density of the emitting matter, which creates the spectral line. If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the emitters, meaning the number of the emitters per square centimetre

$$n = \frac{\sigma}{AW} \left(\text{in } \text{cm}^{-2} \right).$$

This number density corresponds to the energy density which is emitted by the whole matter which creates the observed spectral line

($\frac{E}{AW}$) (in $\frac{\text{erg}}{\text{cm}^2}$) and which is calculated by the model.

It is well known, that each emitter emits the specific amount of the energy needed for the transition which creates the specific line.

This means that if we divide the calculated energy density ($\frac{E}{AW}$) with the energy needed for the transition, we obtain the column density (in cm^{-2}).

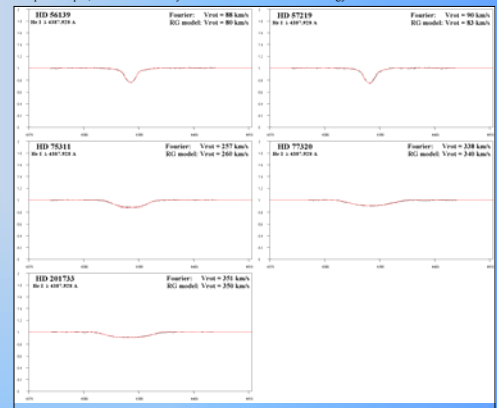
Testing the model

In order to check the above spectral line function, we calculated the rotational velocity of Hel λ 4387.928 Å absorption line in the spectra of five Be stars, using two methods, the classical Fourier analysis and our model. The results are favorable for our model.

The values of the rotational velocities, calculated with Fourier analysis, some time, are a little higher than the values calculated with our method, as in Fourier analysis the whole broadening of the spectral lines is assumed to represent the rotational velocity.

In contrary, our model accept that a part of this broadening arise from the random motion of the ions.

We point out that with our model, apart from the rotational velocities, we can also calculate some other parameters as the standard Gaussian deviation (σ), the velocity of random motions of the ions the radial velocities of the regions producing the studied spectral lines, the full width at half maximum (FWHM), the optical depth, the column density and the absorbed or emitted energy.



The five Hel λ 4387.928 Å fittings for the studied Be stars with RG model and the measured rotational velocities with Fourier analysis and RG model.

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